Chapter 17 – Differentiation and anti-differentiation of polynomials

Solutions to Exercise 17A

1 Average speed $= \dfrac{48-32}{8-3} = \dfrac{16}{5}$ m/s^2

2 a $f(x) = 2x + 5$

Av. rate of change:

$$\frac{f(3)-f(0)}{3-0} = \frac{11-5}{3} = 2$$

b $f(x) = 3x^2 + 4x - 2$

Av. rate of change:

$$\frac{f(2)-f(-1)}{2-(-1)} = \frac{18-(-3)}{3} = \frac{21}{3} = 7$$

c $f(x) = \dfrac{2}{x-3} + 4$

Av. rate of change:

$$\frac{f(7)-f(4)}{7-4} = \frac{4.5-6}{3} = -\frac{1}{2}$$

d $f(x) = \sqrt{5-x}$

Av. rate of change:

$$\frac{f(4)-f(0)}{4-0} = \frac{1-\sqrt{5}}{4}$$

3 a Av. rate of change: $\dfrac{5-30}{2-(-5)} = -\dfrac{25}{7}$

b Av. rate: $\dfrac{5-14}{2-(-1.5)} = -\dfrac{9}{3.5} = -\dfrac{18}{7}$

c Av. rate: $\dfrac{15-3}{3-0} = \dfrac{12}{3} = 4$

d Av. rate: $\dfrac{5b-b}{2a-(-a)} = \dfrac{4b}{3a}$

4 $S(t) = t^3 + t^2 - 2t,\ t > 0$

a Av. rate: $\dfrac{S(2)-S(0)}{2-0} = \dfrac{8}{2} = 4$m/s

b Av. rate: $\dfrac{S(4)-S(2)}{4-2} = \dfrac{72-8}{2}$

$= 32$m/s

5 \$2000 dollars, 7% per year over 3 years

$\therefore I = 2000(1.07^t)$

a $I(3) = 2000(1.07^3) = \$2450.09$

b Av. return $= \dfrac{2450.09-2000}{3} =$

\$150.03

6 $d(t) = -\dfrac{300}{t+6} + 50,\ t > 0$

$d(10) = (50 - \dfrac{300}{16}) = 31.25$ cm

$d(0) = \left(50 - \dfrac{300}{6}\right) = 0$ cm

Av. rate: $\dfrac{31.25}{10} = 3.125$ cm/min

7 C $d(3) = 2$m, $d(0) = 0$m

Av. speed $= \dfrac{2}{3}$m/s

Solutions to Exercise 17B

1 $y = x^3 + x^2$; chord from $x = 1.2$ to 1.3:

$$\cong \frac{y(1.3) - y(1.2)}{1.3 - 1.2} = \frac{3.887 - 3.168}{0.1} = 7.19$$

2 a

From 0 to 1200, av. rate $= \frac{19 - 5}{1200} \approx 0.012$L/kgm

b $C(600) = 15$L/min, $C(0) = 5$L/min.
$W = 450$, est. rate $= \frac{15 - 5}{600} = \frac{1}{60}$
≈ 0.0167L/kg m

3 $y = 10^x$

a Average rate of change over:

i $[0, 1] : \frac{y(1) - y(0)}{1} = \frac{10 - 1}{1} = 9$

ii

$[0, 0.5] : \frac{y(0.5) - y(0)}{0.5} = \frac{\sqrt{10} - 1}{0.5} \cong 4.3246$

iii $[0, 0.1] : \frac{y(0.1) - y(0)}{0.1} \cong 2.5893$

b Even smaller intervals suggest the instantaneous rate of change at $x = 0$ is about 2.30

4 a $T \approx 25°$ at $t = 16$ hours, i.e. at 16:00.

b $T(14) = 23°$, $T(10) = 9°$ (approx.)
Est. rate $= \frac{23 - 10}{14 - 10} \approx 3°$C/hr

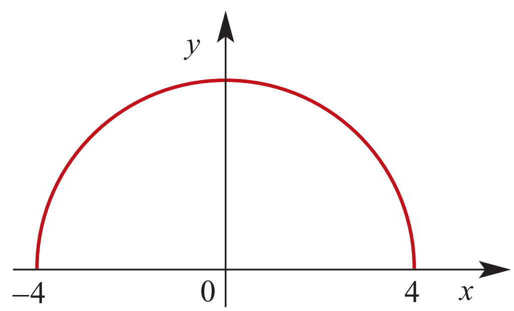
c $T(20) = 15.2°$, $T(16) = 25.2°$

Est. rate $= \frac{15.2 - 25.2}{20 - 16} = -2.5°$C/hr

5 Using chord $x = 1.2$ to 1.4, av. rate of change

$$= \left(\frac{1}{1.4} - \frac{1}{1.2}\right) \div (1.4 - 1.2)$$
$$\cong \frac{0.714 - 0.833}{0.2} = -0.5952$$

6 $y = \sqrt{16 - x^2}$, $-4 \le x \le 4$

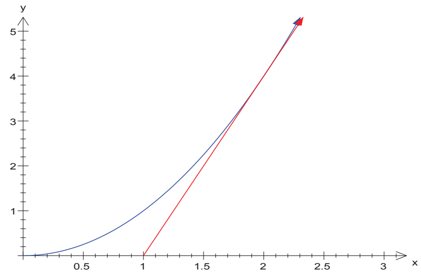


a Gradient at $x = 0$ must be zero, as a tangent drawn at that point is horizontal.

b $x = 2$; chord connecting $x = 1.9$ and 2.1.
$y(2.1) = \sqrt{16 - 2.1^2} \cong 3.40$
$y(1.9) = \sqrt{16 - 1.9^2} \cong 3.52$
Av. rate $= \frac{3.40 - 3.52}{2.1 - 1.9} \cong -0.6$

c $x = 3$; chord connecting $x = 2.9$ and 3.1.
$y(3.1) = \sqrt{16 - 3.1^2} \cong 2.53$
$y(2.9) = \sqrt{16 - 2.9^2} \cong 2.76$
Av. rate $= \frac{2.53 - 2.76}{3.1 - 2.9} \cong -1.1$

7 $y = x^2$ and $y = 4x - 4$:



Graphs meet at (2, 4), where the line is a tangent.
Gradient = 4 (= gradient of $y = 4x - 4$)

8 $V = 3t^2 + 4t + 2$

a Av. rate of change from $t = 1$ to $t = 3$:
$$\frac{V(3) - V(1)}{3 - 1} = \frac{41 - 9}{2}$$
$= 16$
$= 16\ \text{m}^3/\text{min}$

b Est. rate of change at $t = 1$, chord 0.9 to 1.1:
$$\frac{V(1.1) - V(0.9)}{1.1 - 0.9} = \frac{10.03 - 8.03}{0.2}$$
$= 10$
$= 10\ \text{m}^3/\text{min}$

9 $P = 3(2^t)$

a Av. rate of change from $t = 2$ to $t = 4$:
$$\frac{P(4) - P(2)}{4 - 2} = \frac{48 - 12}{2}$$
$= 18$
$= 18$ million/min

b Est. rate of change at $t = 2$, chord 1.9 to 2.1:
$$\frac{P(2.1) - P(1.9)}{2.1 - 1.9} \cong \frac{12.86 - 11.20}{0.2}$$
$= 8.30$
$= 8.30$ million/min

10 $V = 5 \times 10^5 - 10^2(2^t),\ 0 \le t \le 12$

a Av. rate of change from $t = 0$ to $t = 5$:
$$\frac{V(5) - V(0)}{5 - 0} = \frac{-3200 + 100}{5}$$
$= -620\ \text{m}^3/\text{min}$
i.e. 620 m^3/min flowing out

b Est. rate of change at $t = 6$, chord 5.9 to 6.1:
$$\frac{V(1.1) - V(0.9)}{6.1 - 5.9} = \frac{-686 + 597}{0.2}$$
$\cong -4440\ \text{m}^3/\ \text{min}$
i.e. 4440 m^3/min flowing out

c Est. rate of change at $t = 12$, chord 11.9 to 12:
$$\frac{V(12) - V(11.9)}{12 - 11.9} = \frac{-409600 + 382200}{0.1}$$
$\cong -284000\ \text{m}^3/\text{min}$
i.e. 284 000 m^3/min flowing out

11 **a** $y = x^3 + 2x^2$; chord from $x = 1$ to 1.1:
$$\cong \frac{y(1.1) - y(1)}{1.1 - 1} = \frac{3.751 - 3}{0.1}$$
$= 7.51$

b $y = 2x^3 + 3x$;
chord from $x = 1$ to 1. 1:

$$\cong \frac{y(1.1) - y(1)}{1.1 - 1} = \frac{5.962 - 5}{0.1}$$
$$= 9.62$$

c $y = -x^3 + 3x^2 + 2x$;
chord from $x = 2$ to 2.1:

$$\cong \frac{y(2.1) - y(2)}{2.1 - 2} = \frac{8.169 - 8}{0.1}$$
$$= 1.69$$

d $y = 2x^3 - 3x^2 - x + 2$;
chord from $x = 3$ to 3.1:

$$\cong \frac{y(3.1) - y(3)}{3.1 - 3} = \frac{29.7 - 26}{0.1}$$
$$= 37$$

(Using smaller chords give answers which approach $a7, b9, c2, d35$)

12 $V = x^3$

a Av. rate of change from $x = 2$ to $x = 4$:

$$\frac{V(4) - V(2)}{4 - 2} = \frac{64 - 8}{2}$$
$$= 28$$

b Est. rate of change at $t = 2$, chord 1.9 to 2.1:

$$\frac{V(2.1) - V(1.9)}{2.1 - 1.9} \cong \frac{9.261 - 6.859}{0.2}$$
$$= 12.01$$

13 **a** **i** $\frac{2}{\pi} \approx 0.637$

ii $\frac{2\sqrt{2}}{\pi} \approx 0.9003$

iii 0.959

iv 0.998

b 1

Solutions to Exercise 17C

1 a

$$\text{Gradient} = \frac{-(3+h)^2 + 4(3+h) - 3}{3+h-3}$$
$$= \frac{-(9+6h+h^2) + 12 + 4h - 3}{h}$$
$$= \frac{-9 - 6h - h^2 + 12 + 4h - 3}{h}$$
$$= \frac{-2h - h^2}{h}$$
$$= -2 - h$$

b $\lim_{h \to 0}(-2 - h) = -2$

2 a

$$\text{Gradient} = \frac{(4+h)^2 - 3(4+h) - 4}{4+h-4}$$
$$= \frac{16 + 8h + h^2 - 12 - 3h - 4}{h}$$
$$= \frac{5h + h^2}{h}$$
$$= 5 + h$$

b $\lim_{h \to 0}(5 + h) = 5$

3 Gradient

$$= \frac{(x+h)^2 - 2(x+h) - (x^2 - 2x)}{x+h-x}$$
$$= \frac{x^2 + 2xh + h^2 - 2x - 2h - x^2 + 2x}{h}$$
$$= \frac{2xh + h^2 - 2h}{h}$$
$$= 2x + h - 2$$

$\lim_{h \to 0}(2x + h - 2) = 2x - 2$

4 $\text{Gradient} = \frac{(2+h)^4 - 16}{2+h-2}$

$$= \frac{16 + 32h + 24h^2 + h^4 - 16}{h}$$
$$= \frac{32h + 24h^2 + h^4}{h}$$
$$= 32 + 24h + h^3$$

$\lim_{h \to 0}(32 + 24h + h^3) = 32$

5 $V = 3t^2 + 4t + 2$

$$V(1+h) - V(1) = 3(1+h)^2 + 4(1+h) + 2 - 3h^2 - 4h - 2$$
$$= 3h^2 + 10h$$

$$\text{Chord gradient} = \frac{3h^2 + 10h}{1+h-1}$$
$$= 3h + 10$$

Now let $h \to 0$ for the rate of change.
The rate of change of volume at $t = 1$ is $10\,\text{cm}^3/\text{min}$

6 $P = 1000 + t^2 + t,\ t > 0$

$$P(3+h) - P(3) = (3+h)^2 - 9 + (3+h) - 3$$
$$= 6h + h^2 + h$$
$$= 7h + h^2$$

$$\text{Chord gradient} = \frac{7h + h^2}{3+h-3}$$
$$= 7 + h$$

Growth rate at $t = 3$ is 7 insects/day

7 a $\lim_{h\to 0} \frac{2x^2h^3 + xh^2 + h}{h}$

$= \lim_{h\to 0} 2x^2h^2 + xh + 1 = 1$

b $\lim_{h\to 0} \frac{3x^2h - 2xh^2 + h}{h}$

$= \lim_{h\to 0} 3x^2 - 2xh + 1 = 3x^2 + 1$

c $\lim_{h\to 0} 20 - 10h = 20$

d $\lim_{h\to 0} \frac{30hx^2 + 2h^2 + h}{h}$

$= \lim_{h\to 0} 30x^2 + 2h + 1 = 30x^2 + 1$

e $\lim_{h\to 0} 5 = 5$

f $\lim_{h\to 0} \frac{30hx^3 + 2h^2 + 4h}{h} =$

$\lim_{h\to 0} 30x^3 + 2h + 4 = 30x^3 + 4$

8 a $\lim_{h\to 0} \frac{(x+h)^2 + 2(x+h) - (x^2 + 2x)}{h}$

$= \lim_{h\to 0} \frac{2hx + h^2 + 2h}{h} = 2x + 2$

b $\lim_{h\to 0} \frac{(5+h)^2 + 3(5+h) - 40}{h}$

$= \lim_{h\to 0} \frac{10h + h^2 + 3h}{h}$

$= \lim_{h\to 0} 13 + h = 13$

c $\lim_{h\to 0} \frac{(x+h)^3 + 2(x+h)^2 - (x^3 + 2x^2)}{h}$

$= \lim_{h\to 0} \frac{3x^2h + 3xh^2 + h^3 + 4xh + 2h^2}{h}$

$= \lim_{h\to 0} 3x^2 + 3xh + h^2 + 4x + 2h$

$= 3x^2 + 4x$

9 $y = 3x^2 - x$

a Gradient of chord PQ:

$= \frac{3(1+h)^2 - (1+h) - 2}{1 + h - 1}$

$= \frac{3(1 + 2h + h^2) - 1 - h - 2}{h}$

$= \frac{6h + 3h^2 - h}{h} = 5 + 3h$

b Gradient of PQ when $h = 0.1$ is 5.3

c Gradient of the curve at $P = 5$

10 $y = \frac{2}{x}$

a Gradient of chord AB:

$= \frac{\frac{2}{2+h} - 1}{2 + h - 2}$

$= \frac{2 - (2+h)}{h(2+h)}$

$= \frac{-h}{h(2+h)} = \frac{-1}{2+h}$

b Gradient of AB when $h = 0.1 \cong -0.48$

c Gradient of the curve at $A = -\frac{1}{2}$

11 $y = x^2 + 2x - 3$

a Gradient of chord PQ:

$= \frac{(2+h)^2 + 2(2+h) - 3 - 5}{2 + h - 2}$

$= \frac{4 + 4h + h^2 + 4 + 2h - 8}{h}$

$= \frac{6h + h^2}{h} = 6 + h$

b Gradient of PQ when $h = 0.1$ is 6.1

c Gradient of the curve at $P = 6$

12 Derivatives from first principles

a $\lim\limits_{h\to 0} \dfrac{3(x+h)^2 - 3x^2}{h}$
$= \lim\limits_{h\to 0} \dfrac{3x^2 + 6xh + 3h^2 - 3x^2}{h}$
$= \lim\limits_{h\to 0} \dfrac{6xh + 3h^2}{h}$
$= \lim\limits_{h\to 0} 6x + 3h = 6x$

b $\lim\limits_{h\to 0} \dfrac{4(x+h) - 4x}{h}$
$= \lim\limits_{h\to 0} \dfrac{4h}{h} = 4$

c $\lim\limits_{h\to 0} \dfrac{3-3}{h} = 0$

d
$\lim\limits_{h\to 0} \dfrac{3(x+h)^2 + 4(x+h) - 3 - 3x^2 - 4x + 3}{h}$
$= \lim\limits_{h\to 0} \dfrac{6hx + 3h^2 + 4h}{h}$
$= \lim\limits_{h\to 0} 6x + 3h + 4 = 6x + 4$

e $\lim\limits_{h\to 0} \dfrac{2(x+h)^3 - 4 - 2x^3 + 4}{h}$
$= \lim\limits_{h\to 0} \dfrac{6x^2h + 6xh^2 + 2h^3}{h}$
$= \lim\limits_{h\to 0} 6x^2 + 6xh + 2h^2 = 6x^2$

f $\lim\limits_{h\to 0} \dfrac{4(x+h)^2 - 5(x+h) - 4x^2 + 5x}{h}$
$= \lim\limits_{h\to 0} \dfrac{4x^2 + 8hx + 4h^2 - 5x - 5h - 4x^2 + 5x}{h}$
$= \lim\limits_{h\to 0} \dfrac{8hx + 4h^2 - 5h}{h}$
$= \lim\limits_{h\to 0} 8x + 4h - 5 = 8x - 5$

g
$\lim\limits_{h\to 0} \dfrac{3 - 2(x+h) + (x+h)^2 - 3 + 2x - x^2}{h}$
$= \lim\limits_{h\to 0} \dfrac{-2x - 2h + x^2 + 2hx + h^2 + 2x - x^2}{h}$
$= \lim\limits_{h\to 0} \dfrac{-2h + 2hx + h^2}{h}$
$= \lim\limits_{h\to 0} -2 + 2x + h = 2x - 2$

13 Gradient
$= \dfrac{(x+h)^4 - x^4}{x + h - x}$
$= \dfrac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h}$
$= \dfrac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$
$= 4x^3 + 6x^2h + 4xh^2 + h^3$
$\lim\limits_{h\to 0}(4x^3 + 6x^2h + 4xh^2 + h^3) = 4x^3$

Solutions to Exercise 17D

1 Derivatives using $\frac{d}{dx}x^n = nx^{n-1}$

a $\frac{d}{dx}(x^2 + 4x) = 2x + 4$

b $\frac{d}{dx}(2x + 1) = 2$

c $\frac{d}{dx}(x^3 - x) = 3x^2 - 1$

d $\frac{d}{dx}\left(\frac{1}{2}x^2 - 3x + 4\right) = x - 3$

e $\frac{d}{dx}(5x^3 + 3x^2) = 15x^2 + 6x$

f $\frac{d}{dx}(-x^3 + 2x^2) = -3x^2 + 4x$

2 **a** $f(x) = x^{12}, \therefore f'(x) = 12x^{11}$

b $f(x) = 3x^7, \therefore f'(x) = 21x^6$

c $f(x) = 5x, \therefore f'(x) = 5$

d $f(x) = 5x + 3, \therefore f'(x) = 5$

e $f(x) = 3, \therefore f'(x) = 0$

f $f(x) = 5x^2 - 3x, \therefore f'(x) = 10x - 3$

g $f(x) = 10x^5 + 3x^4,$
$\therefore f'(x) = 50x^4 + 12x^3$

h $f(x) = 2x^4 - \frac{1}{3}x^3 - \frac{1}{4}x^2 + 2$
$\therefore f'(x) = 8x^3 - x^2 - \frac{1}{2}x$

3 **a** $f(x) = x^6, \therefore f'(x) = 6x^5, \therefore f'(1) = 6$

b $f(x) = 4x^5, \therefore f'(x) = 20x^4,$
$\therefore f'(1) = 20$

c $f(x) = 5x, \therefore f'(x) = 5, \therefore f'(1) = 5$

d $f(x) = 5x^2 + 3, \therefore f'(x) = 10x,$
$\therefore f'(1) = 10$

e $f(x) = 3, \therefore f'(x) = 0, \therefore f'(1) = 0$

f $f(x) = 5x^2 - 3x, \therefore f'(x) = 10x - 3,$
$\therefore f'(1) = 7$

g $f(x) = 10x^4 - 3x^3,$
$\therefore f'(x) = 40x^3 - 9x^2, \therefore f'(1) = 31$

h $f(x) = 2x^4 - \frac{1}{3}x^3, \therefore f'(x) = 8x^3 - x^2,$
$\therefore f'(1) = 7$

i $f(x) = -10x^3 - 2x^2 + 2,$
$\therefore f'(x) = -30x^2 - 4x, \therefore f'(1) = -34$

4 **a** $f(x) = 5x^3, \therefore f'(x) = 15x^2,$
$\therefore f'(-2) = 60$

b $f(x) = 4x^2, \therefore f'(x) = 8x,$
$\therefore f'(-2) = -16$

c $f(x) = 5x^3 - 3x, \therefore f'(x) = 15x^2 - 3,$
$\therefore f'(-2) = 57$

d $f(x) = -5x^4 - 2x^2,$
$\therefore f'(x) = -20x^3 - 4x,$
$\therefore f'(-2) = 168$

5 **a** $f(x) = x^2 + 3x, \therefore f'(x) = 2x + 3,$
$\therefore f'(2) = 7$

b $f(x) = 3x^2 - 4x, \therefore f'(x) = 6x - 4,$
$\therefore f'(1) = 2$

c $f(x) = -2x^2 - 4x, \therefore f'(x) = -4x - 4,$
$\therefore f'(3) = -16$

d $f(x) = x^3 - x, \therefore f'(x) = 3x^2 - 1,$
$\therefore f'(2) = 11$

6 a $y = -x, \therefore \frac{dy}{dx} = -1$

b $y = 10, \therefore \frac{dy}{dx} = 0$

c $y = 4x^3 - 3x + 2, \therefore \frac{dy}{dx} = 12x^2 - 3$

d $y = \frac{1}{3}(x^3 - 3x + 6)$
$= \frac{1}{3}x^3 - x + 2$
$\therefore \frac{dy}{dx} = x^2 - 1$

e $y = (x + 1)(x + 2)$
$= x^2 + 3x + 2$
$\therefore \frac{dy}{dx} = 2x + 3$

f $y = 2x(3x^2 - 4)$
$= 6x^3 - 8x$
$\therefore \frac{dy}{dx} = 18x^2 - 8$

g $y = \frac{10x^5 + 3x^4}{2x^2}$
$= 5x^3 + \frac{3}{2}x^2, \ x \neq 0$
$\therefore \frac{dy}{dx} = 15x^2 + 3x$

7 a $y = (x + 4)^2 = x^2 + 8x + 16$
$\frac{dy}{dx} = 2x + 8$

b $z = (4t - 1)^2(t + 1)$
$= (16t^2 - 8t + 1)(t + 1)$
$= 16t^3 - 8t^2 + t + 16t^2 - 8t + 1$
$= 16t^3 + 8t^2 - 7t + 1$
$\therefore \frac{dz}{dt} = 48t^2 + 16t - 7$

c $\frac{x^3 + 3x}{x} = x^2 + 3 \therefore \frac{dy}{dx} = 2x$

8 a $y = x^3 + 1, \therefore \frac{dy}{dx} = 3x^2$

i Gradient at $(1, 2) = 3$

ii Gradient at $(a, a^3 + 1) = 3a^2$

b Derivative $= 3x^2$

9 a $y = x^3 - 3x^2 + 3x$
$\therefore \frac{dy}{dx} = 3x^2 - 6x + 3$
$= 3(x + 1)^2 \geq 0$
The graph of $y = x^3 - 3x^2 + 3x$ will have a positive gradient for all x, except for a saddle point at $x = -1$ where the gradient = 0.

b $y = \frac{x^2 + 2x}{x} = x + 2, \ x \neq 0$
$\therefore \frac{dy}{dx} = 1, \ x \neq 0$

c $y = (3x + 1)^2 = 9x^2 + 6x + 1$
$\therefore \frac{dy}{dx} = 18x + 6 = 6(3x + 1)$

10 a $y = x^2 - 2x + 1, \therefore \frac{dy}{dx} = 2x - 2$
$\therefore y(2) = 1, \ y'(2) = 2$

b $y = x^2 + x + 1, \therefore \frac{dy}{dx} = 2x + 1$
$\therefore y(0) = 1, \ y'(0) = 1$

c $y = x^2 - 2x, \therefore \frac{dy}{dx} = 2x - 2$
$\therefore y(-1) = 3, \ y'(-1) = -4$

d $y = (x + 2)(x - 4) = x^2 - 2x - 8$

$\therefore \dfrac{dy}{dx} = 2x - 2$

$\therefore y(3) = -5,\ y'(3) = 4$

e $y = 3x^2 - 2x^3, \therefore \dfrac{dy}{dx} = 6x - 6x^2$

$\therefore y(-2) = 28,\ y'(-2) = -36$

f $y = (4x - 5)^2 = 16x^2 - 40x + 25$

$\therefore \dfrac{dy}{dx} = 32x - 40 = 8(4x - 5)$

$\therefore y\left(\dfrac{1}{2}\right) = 9, y'\left(\dfrac{1}{2}\right) = -24$

11 a i $f(x) = 2x^2 - x,\ \therefore f'(x) = 4x - 1$

$\therefore f'(1) = 3$

Gradient = 1 when $4x - 1 = 1$

$\therefore x = \dfrac{1}{2}$ and $f\left(\dfrac{1}{2}\right) = 0$

Gradient = 1 at $\left(\dfrac{1}{2}, 0\right)$

ii $f(x) = 1 + \dfrac{1}{2}x + \dfrac{1}{3}x^2$

$\therefore f'(x) = \dfrac{2}{3}x + \dfrac{1}{2},\ \therefore f'(1) = \dfrac{7}{6}$

Gradient = 1 when $\dfrac{2}{3}x + \dfrac{1}{2} = 1$

$\therefore x = \dfrac{1}{2}\left(\dfrac{3}{2}\right) = \dfrac{3}{4}$ and $f\left(\dfrac{3}{4}\right) = \dfrac{25}{16}$

Gradient = 1 at $\left(\dfrac{3}{4}, \dfrac{25}{16}\right)$

iii $f(x) = x^3 + x, \therefore f'(x) = 3x^2 + 1$

$\therefore f'(1) = 4$

Gradient = 1 when $3x^2 + 1 = 1$

$\therefore x = 0$ and $f(0) = 0$

Gradient = 1 at $(0, 0)$

iv $f(x) = x^4 - 31x$,

$\therefore f'(x) = 4x^3 - 31$

$\therefore f'(1) = -27$

Gradient = 1 when $4x^3 - 31 = 1$

$\therefore 4x^3 = 32$

$\therefore x = 2$ and $f(2) = -46$

Gradient = 1 at $(2, -46)$

b Points where the gradients equal 1 are where a tangent makes an angle of 45° to the axes.

12 a $\dfrac{d}{dt}(3t^2 - 4t) = 6t - 4$

b $\dfrac{d}{dx}(4 - x^2 + x^3) = -2x + 3x^2$

c $\dfrac{d}{dz}(5 - 2z^2 - z^4) = -4z - 4z^3$

$= -4z(z^2 + 1)$

d $\dfrac{d}{dy}(3y^2 - y^3) = 6y - 3y^2$

$= 3y(2 - y)$

e $\dfrac{d}{dx}(2x^3 - 4x^2) = 6x^2 - 8x$

$= 2x(3x - 4)$

f $\dfrac{d}{dt}(9.8t^2 - 2t) = 19.6t - 2$

13 a $y = x^2,\ \therefore \dfrac{dy}{dx} = 2x$

Gradient = 8 at $(4, 16)$

b $y = x^3,\ \therefore \dfrac{dy}{dx} = 3x^2 = 12,\ \therefore x = \pm 2$

Gradient = 12 at $(-2, -8), (2, 8)$

c $y = x(2 - x) = 2x - x^2,\ \therefore \dfrac{dy}{dx} = 2 - 2x$

Gradient = 2 where $x = 0$, i.e. at $(0, 0)$

d $y = x^2 - 3x + 1,\ \therefore \dfrac{dy}{dx} = 2x - 3$

Gradient = 0 where $x = \dfrac{3}{2}$, i.e. at $\left(\dfrac{3}{2}, -\dfrac{5}{4}\right)$

e $y = x^3 - 6x^2 + 4,\ \dfrac{dy}{dx} = 3x^2 - 12x$

Gradient = -12 where

$3x^2 - 12x + 12 = 0$
$\therefore x^2 - 4x + 4 = 0$

$(x-2)^2 = 0, \ \therefore x = 2$
i.e. at $(2, -12)$

f $y = x^2 - x^3 \ \therefore \dfrac{dy}{dx} = 2x - 3x^2$
Gradient $= -1$ where
$-3x^2 + 2x + 1 = 0$
$\therefore 3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$x = -\dfrac{1}{3}, 1$
i.e. at $\left(-\dfrac{1}{3}, \dfrac{4}{27}\right)$ and $(1, 0)$

Solutions to Exercise 17E

1 a

$$\frac{f(x+h)-f(x)}{x+h-x} = \frac{\frac{1}{x+h-3} - \frac{1}{x-3}}{x+h-x}$$

$$= \frac{\frac{(x-3-(x+h-3)}{x+h-3)(x-3)}}{h}$$

$$= \frac{-h}{(x+h-3)(x-3)} \times \frac{1}{h}$$

$$= \frac{-1}{(x+h-3)(x-3)}$$

$$\lim_{h\to 0} \frac{-1}{(x+h-3)(x-3)} = -\frac{1}{(x-3)^2}$$

b

$$\frac{f(x+h)-f(x)}{x+h-x} = \frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{x+h-x}$$

$$= \frac{\frac{(x+2-(x+h+2)}{x+h+2)(x+2)}}{h}$$

$$= \frac{-h}{(x+h+2)(x+2)} \times \frac{1}{h}$$

$$= \frac{-1}{(x+h+2)(x+2)}$$

$$\lim_{h\to 0} \frac{-1}{(x+h+2)(x+2)} = -\frac{1}{(x+2)^2}$$

2 a

$$\frac{f(x+h)-f(x)}{x+h-x} = \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{x+h-x}$$

$$= \frac{\frac{x^2-(x+h)^2}{x+h)^2x^2}}{h}$$

$$= \frac{x^2-x^2-2xh-h^2}{(x+h)^2x^2} \times \frac{1}{h}$$

$$= \frac{-2xh-h^2}{(x+h)^2x^2} \times \frac{1}{h}$$

$$= \frac{-2x-h}{(x+h)^2x^2}$$

$$\lim_{h\to 0} \frac{-2x-h}{(x+h)^2x^2} = -\frac{2}{x^3}$$

b

$$\frac{f(x+h)-f(x)}{x+h-x} = \frac{\frac{1}{(x+h)^4} - \frac{1}{x^4}}{x+h-x}$$

$$= \frac{\frac{x^2-(x+h)^4}{x+h)^2x^2}}{h}$$

$$= \frac{x^4-(x^4+4x^3h+6x^2h^2+4xh^3+h^4}{(x+h)^2x^2} \times \frac{1}{h}$$

$$= \frac{-(4x^3h+6x^2h^2+4xh^3+h^4)}{(x+h)^4x^4} \times \frac{1}{h}$$

$$= \frac{-(4x^3+6xh+4xh^2+h^3)}{(x+h)^4x^4}$$

$$\lim_{h\to 0} \frac{-(4x^3+6xh+4xh^2+h^3)}{(x+h)^4x^4} = -\frac{4x^3}{x^8}$$

$$= -\frac{4}{x^5}$$

3 a $\frac{d}{dx}(3x^{-2}+5x^{-1}+6) = -6x^{-3} - 5x^{-2}$

b $\frac{d}{dx}\left(\frac{3}{x^2}+5x^2\right) = -\frac{6}{x^3} + 10x$

c $\frac{d}{dx}\left(\frac{5}{x^3}+\frac{4}{x^2}+1\right) = -\frac{15}{x^4} - \frac{8}{x^3}$

d $\frac{d}{dx}\left(3x^2+\frac{5}{3}x^{-4}+2\right) = 6x - \frac{20}{3}x^{-5}$

e $\frac{d}{dx}(6x^{-2}+3x) = -12x^{-3} + 3$

f
$$\frac{d}{dx}\frac{3x^2+2}{x} = \frac{d}{dx}\left(3x+\frac{2}{x}\right)$$

$$= 3 - \frac{2}{x^2}$$

4 $z \neq 0$ throughout

a $\frac{d}{dz}\frac{3z^2+2z+4}{z^2} = \frac{d}{dz}\left(3+\frac{2}{z}+\frac{4}{z^2}\right)$
$= -\frac{2}{z^2} - \frac{8}{z^3}$

b $\frac{d}{dz}\frac{3+z}{z^3} = \frac{d}{dz}\left(\frac{3}{z^3}+\frac{1}{z^2}\right)$
$= -\frac{9}{z^4} - \frac{2}{z^3}$

c $\frac{d}{dz}\frac{2z^2+3z}{4z} = \frac{d}{dz}\left(\frac{z}{2}+\frac{3}{4}\right) = \frac{1}{2}$

d $\frac{d}{dz}(9z^2+4z+6z^{-3}) = 18z+4-18z^{-4}$

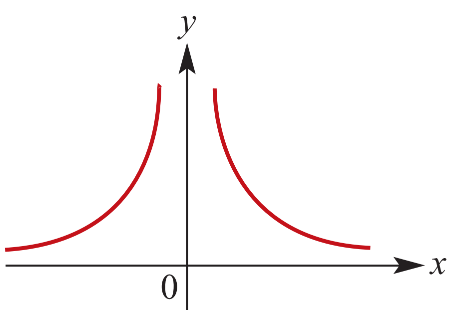
e $\frac{d}{dz}(9-z^{-2}) = -2z^{-3}$

f $\frac{d}{dz}\frac{5z-3z^2}{5z} = \frac{d}{dz}\left(5-\frac{3z}{5}\right) = -\frac{3}{5}$

5 **a** $f'(x) = 12x^3+18x^{-4}-x^{-2}$

b $f'(x) = 20x^3-8x^{-3}-x^{-2}$

6 $f(x) = \frac{1}{x^2}; x \neq 0$



a $P = (1, f(1)); Q = (1+h, f(1+h))$

$PQ's \text{ gradient } = \frac{\frac{1}{(1+h)^2}-1}{h}$
$= \frac{1-1-2h-h^2}{h(1+h)^2}$
$= \frac{-2-h}{(1+h)^2}$

b $f(x) = \frac{1}{x^2}$ has gradient of -2 at $x = 1$

7 **a** $y = x^{-2}+x^3, \therefore\ y' = -2x^{-3}+3x^2$
$\therefore\ y'(2) = -\frac{2}{8}+12 = \frac{47}{4}$

b $y = \frac{x-2}{x} = 1-\frac{2}{x}, \therefore\ y' = \frac{2}{x^2}$
$\therefore y'(4) = \frac{2}{16} = \frac{1}{8}$

c $y = x^{-2}-\frac{1}{x}, \therefore\ y' = -\frac{2}{x^3}+\frac{1}{x^2}$
$\therefore\ y'(1) = -2+1 = -1$

d $y = x(x^{-1}+x^2-x^{-3}) = 1+x^3-x^{-2}$
$\therefore\ y' = 3x^2+2x^{-3}$
$\therefore\ y'(1) = 3+2 = 5$

8 $f(x) = x^{-2},\ \therefore\ f'(x) = -2x^{-3}; x > 0$

a $f'(x) = -2x^{-3} = 16,\ \therefore\ x^3 = -\frac{1}{8}$
$\therefore\ x = -\frac{1}{2}$

b $f'(x) = -2x^3 = -16, \therefore\ x^3 = \frac{1}{8}$
$\therefore x = \frac{1}{2}$

9 $f'(x) = -x^{-2} = -\frac{1}{x^2} < 0$ for all non-zero x

Solutions to Exercise 17F

1 $\frac{dy}{dx}$ = gradient

a $\frac{dy}{dx} < 0$ for all x

b $\frac{dy}{dx} > 0$ for all x

c $\frac{dy}{dx}$ varies in sign

d $\frac{dy}{dx} > 0$ for all x

e $\frac{dy}{dx} > 0$ for all $x > 0$ and $\frac{dy}{dx} < 0$ for all $x < 0$

Gradient is uniformly positive for **b** and **d** only.

2 $\frac{dy}{dx}$ = gradient:

a $\frac{dy}{dx} < 0$ for all x

b $\frac{dy}{dx} < 0$ for all x

c $\frac{dy}{dx}$ varies in sign

d $\frac{dy}{dx}$ varies in sign

e $\frac{dy}{dx} < 0$ for all x

f $\frac{dy}{dx} = 0$ for all x

Gradient is uniformly negative for **a**, **b** and **e** only.

3 $f(x) = 2(x-1)^2$

a $f(x) = 0, \therefore 2(x-1)^2 = 0$

$x = 1$

b $f'(x) = 4x - 4 = 0, \therefore x = 1$

c $f'(x) = 4x - 4 > 0, \therefore x > 1$

d $f'(x) = 4x - 4 < 0, \therefore x < 1$

e $f'(x) = 4x - 4 = -2$

$4x = 2, \therefore x = \frac{1}{2}$

4 a $\{x: h'(x) > 0\}$

$= \{x: x < -3\} \cup \{x: \frac{1}{2} < x < 4\}$

b $\{x: h'(x) < 0\}$

$= \{x: -3 < x < \frac{1}{2}\} \cup \{x: x > 4\}$

c $\{x: h'(x) = 0\} = \{-3, \frac{1}{2}, 4\}$

5 a $\frac{dy}{dx} < 0$ for $x < 0$, $\frac{dy}{dx} = 0$ at $x = 0$,

$\frac{dy}{dx} > 0$ for $x > 0$

$\therefore \frac{dy}{dx}$ = line $y = kx,\ k > 0$ **B**

b $\frac{dy}{dx} > 0$ for $x < 0$ and $x > a > 0$,

$\frac{dy}{dx} = 0$ at $x = 0$ and a, $\frac{dy}{dx} < 0$ for $0 < x < a$

$\therefore \frac{dy}{dx}$ = is a curve like a parabola $y = kx(x-a)$ **C**

c $\frac{dy}{dx} < 0$ for all x except $\frac{dy}{dx} = 0$ at $x = 0$ and a, **D**

d $\frac{dy}{dx} > 0$ for $x < a > 0$, $\frac{dy}{dx} = 0$ at $x = a$,
$\frac{dy}{dx} > 0$ for $x > a$, **A**

e $y = -k,\ k > 0$ for all x so $\frac{dy}{dx} = 0$ **F**

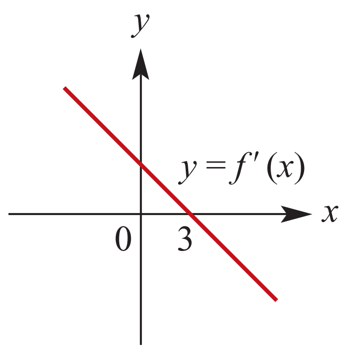
f $y = kx + c;\ k,\ c > 0$ so $\frac{dy}{dx} = k$ **E**

6 a $\{x\colon f'(x) > 0\} = \{x\colon x < 3\}$

$\{x\colon f'(x) < 0\} = \{x\colon x > 3\}$

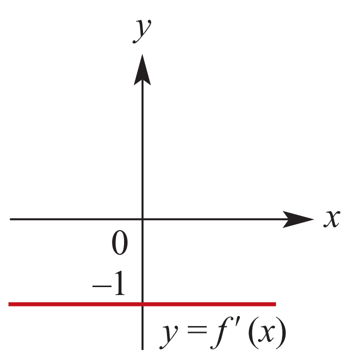
$\{x\colon f'(x) = 0\} = \{3\}$

$\therefore f'(x) = -k(x-3),\ k > 0$



b $f(x) = 1 - x$

$\therefore f'(x) = -1$



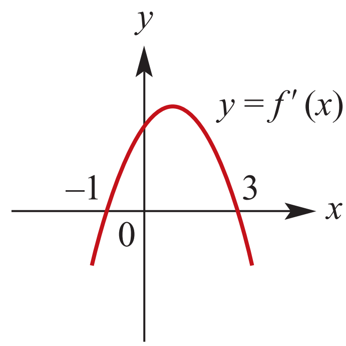
c

$\{x\colon f'(x) > 0\} = \{x\colon -1 < x < 3\}$

$\{x\colon f'(x) < 0\} = \{x\colon x < -1\} \cup \{x\colon x > 3\}$

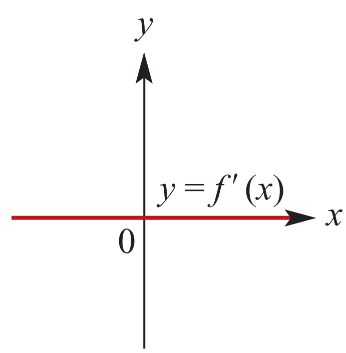
$\{x\colon f'(x) = 0\} = \{-1, 3\}$

$\therefore f'(x) = -k(x-3)(x+1),\ k > 0$



d $f(x) = 3$

$\therefore f'(x) = 0$



7 a $\{x\colon f'(x) > 0\} = \{x\colon -1 < x < 1.5\}$

b $\{x\colon f'(x) < 0\}$
$= \{x\colon x < -1\} \cup \{x\colon x > 1.5\}$

c $\{x\colon f'(x) = 0\} = \{-1, 1.5\}$

8 $y = x^2 - 5x + 6, \therefore \frac{dy}{dx} = 2x - 5$

a Tangent makes an angle of 45° with the positive direction of the x-axis
$\therefore$ gradient = 1
$\therefore \frac{dy}{dx} = 2x - 5 = 1, \therefore x = 3$
$y(3) = 0$ so coordinates are $(3, 0)$.

b Tangent parallel to $y = 3x + 4$

$\therefore$ gradient $= 3$

$\therefore \dfrac{dy}{dx} = 2x - 5 = 3, \therefore x = 4$

$y(4) = 2$ so coordinates are $(4, 2)$.

9 $y = x^2 - x - 6, \therefore \dfrac{dy}{dx} = 2x - 1$

a $\dfrac{dy}{dx} = 2x - 1 = 0, \therefore x = \dfrac{1}{2}$

$y\left(\dfrac{1}{2}\right) = -\dfrac{25}{4}$ so coordinates are

$\left(\dfrac{1}{2}, -\dfrac{25}{4}\right)$.

b Tangent parallel to $x + y = 6$

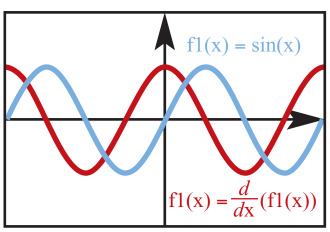
$\therefore$ gradient $= -1$

$\therefore \dfrac{dy}{dx} = 2x - 1 = -1, \therefore x = 0$

$y(0) = -6$ so coordinates are $(0, -6)$.

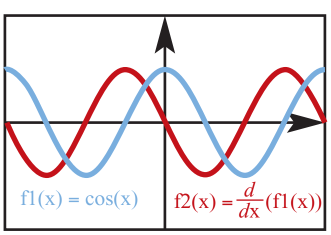
10 **a** $f(x) = \sin x$

$f'(x)$



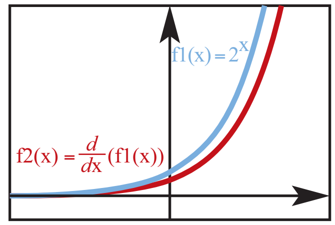
b $f(x) = \cos x$

$f'(x)$



c $f(x) = 2^x$

$f'(x)$



11 **a** **i** 66.80°

ii 42.51°

b $(0.5352, 0.2420)$

c No

12 $y = ax^2 + bx$

a $y(2) = -2, \therefore 4a + 2b = -2$

$y'(x) = 2ax + b$

$\therefore y'(2) = 4a + b = 3$

$\therefore b = -5, \; a = 2$

b $\dfrac{dy}{dx} = 4x - 5 = 0, \therefore x = \dfrac{5}{4}$

$y\left(\dfrac{5}{4}\right) = 4\left(\dfrac{5}{4}\right)^2 - 5\left(\dfrac{5}{4}\right)$

$= \dfrac{25}{4} - \dfrac{25}{8} = -\dfrac{25}{8}$

Coordinates are $\left(\dfrac{5}{4}, -\dfrac{25}{8}\right)$.

Solutions to Exercise 17G

1 a $\int \frac{1}{2}x^3 dx = \frac{1}{8}x^4 + c$

b $\int 3x^2 - 2dx = x^3 - 2x + c$

c $\int 5x^3 - 2xdx = \frac{5}{4}x^4 - x^2 + c$

d $\int \frac{4}{5}x^3 - 2x^2dx = \frac{1}{5}x^4 - \frac{2}{3}x^3 + c$

e $\int (x-1)^2 dx = \int x^2 - 2x + 1\,dx$

$= \frac{x^3}{3} - x^2 + x + c$

f $\int x\left(x + \frac{1}{x}\right)dx = \int x^2 + 1\,dx$

$= \frac{1}{3}x^3 + x + c$

g $\int 2z^2(z-1)dz = \int 2z^3 - 2z^2 dz$

$= \frac{1}{2}z^4 - \frac{2}{3}z^2 + c$

h $\int (2t-3)^2 dt = \int 4t^2 - 12t + 9\,dt$

$= \frac{4t^3}{3} - 6t^2 + 9t + c$

i $\int (t-1)^3 dt = \int t^3 - 3t^2 + 3t - 1\,dt$

$= \frac{t^4}{4} - t^3 + \frac{3t^2}{2} - t + c$

2 $f'(x) = 4x^3 + 6x^2 + 2$

$\therefore f(x) = x^4 + 2x^3 + 2x + c$

We have, $f(0) = 0$

$\therefore c = 0$

$\therefore f(x) = x^4 + 2x^3 + 2x$

3 $f'(x) = 6x^2$

$\therefore f(x) = 2x^3 + c$

We have, $f(0) = 12$

$\therefore c = 12$

$\therefore f(x) = 2x^3 + 12$

4 a $\frac{dy}{dx} = 2x - 1, \therefore y = x^2 - x + c$

$y(1) = c = 0, \therefore y = x^2 - x$

b $\frac{dy}{dx} = 3 - x, \therefore y = 3x - \frac{1}{2}x^2 + c$

$y(0) = c = 1, \therefore y = -\frac{1}{2}x^2 + 3x + 1$

c $\frac{dy}{dx} = x^2 + 2x, \therefore y = \frac{1}{3}x^3 + x^2 + c$

$y(0) = c = 2, \therefore y = \frac{1}{3}x^3 + x^2 + 2$

d

$\frac{dy}{dx} = 3 - x^2, \therefore y = 3x - \frac{1}{3}x^3 + c$

$y(3) = c = 2, \therefore y = -\frac{1}{3}x^3 + 3x + 2$

e $\frac{dy}{dx} = 2x^4 + x, \therefore y = \frac{2}{5}x^5 + \frac{1}{2}x^2 + c$

$y(0) = c = 0, \therefore y = \frac{2}{5}x^5 + \frac{1}{2}x^2$

5 $\frac{dV}{dt} = t^2 - t,\ t > 1$

a $V(t) = \frac{1}{3}t^3 - \frac{1}{2}t^2 + c$

$V(3) = 9 - \frac{9}{2} + c = 9$

$c = \frac{9}{2}$

b $V(10) = \frac{1000}{3} - \frac{100}{2} + \frac{9}{2}$

$= \frac{1727}{6} \approx 287.833$

6 $f'(x) = 3x^2 - 1, \therefore f(x) = x^3 - x + c$

$f(1) = c = 2, \therefore f(x) = x^3 - x + 2$

7 **a** Only **B** has the correct gradient (negative) with the correct axis intercept.

b $\frac{dw}{dt} = 2000 - 20t, \ t > 0$

$w = 2000t - 10t^2 + c, t \geq 0$

$w(0) = c = 100\,000$

$\therefore w = -10t^2 + 2000t + 100\,000$

8 $\frac{dy}{dx} = 5 - x, \therefore f(x) = 5x - \frac{1}{2}x^2 + c$

$f(0) = c = 4, \therefore f(x) = -\frac{1}{2}x^2 + 5x + 4$

9 $f(x) = x^2(x-3) = x^3 - 3x^2$

$\therefore f(x) = \frac{1}{4}x^4 - x^3 + c$

$f(2) = 4 - 8 + c = -6, \therefore c = -2$

$\therefore f(x) = \frac{1}{4}x^4 - x^3 - 2$

10 $f'(x) = 4x + k, \therefore f(x) = 2x^2 + kx + c$

a $f'(-2) = -8 + k = 0$

$k = 8$

b $f(-2) = 8 - 16 + c = -1, \therefore c = 7$

$\therefore f(x) = 2x^2 + 8x + 7$

$\therefore f(0) = 7$

Curve meets y-axis at $(0, 7)$

11 $\frac{dy}{dx} = ax^2 + 1, \therefore y = \frac{a}{3}x^3 + x + c$

$y'(1) = a + 1 = 3, \therefore a = 2$

$y(1) = \frac{2}{3} + 1 + c = 3, \therefore c = \frac{4}{3}$

$\therefore y(2) = \frac{2}{3}(2)^3 + 2 + \frac{4}{3}$

$= \frac{26}{3}$

12 $\frac{dy}{dx} = 2x + k, \therefore y'(3) = 6 + k$

a Tangent: $y - 6 = (6 + k)(x - 3)$

$y = (6 + k)x - 12 - 3k$

Tangent passes through $(0, 0)$,

$\therefore k = -4$

b $y = \int 2x - 4dx = x^2 - 4x + c$

$y(3) = 9 - 12 + c = 6, \therefore c = 9$

$\therefore y = x^2 - 4x + 9$

13 $f'(x) = 16x + k$

a $y'(2) = 32 + k = 0$

$k = -32$

b $f(x) = \int 16x - 32dx$

$= 8x^2 - 32x + c$

$f(2) = 32 - 64 + c = 1, \therefore c = 33$

$\therefore f(7) = 8(7)^2 - 32(7) + 33$

$= 201$

14 $f'(x) = x^2, \therefore f(x) = \frac{1}{3}x^3 + c$

$f(2) = \frac{8}{3} + c = 1, \therefore c = -\frac{5}{3}$

$\therefore f(x) = \frac{1}{3}(x^3 - 5)$

Solutions to Exercise 17H

1 a $\lim_{x\to 3} 15 = 15$

b $\lim_{x\to 6} (x-5) = 6-5 = 1$

c $\lim_{x\to \frac{1}{2}} (3x-5) = \frac{3}{2} - 5 = -\frac{7}{2}$

d $\lim_{t\to -3} \frac{t-2}{t+5} = \frac{-3-2}{-3+5} = -\frac{5}{2}$

e $\lim_{t\to -1} \frac{t^2+2t+1}{t+1} = \frac{(t+1)^2}{t+1}$
$= \lim_{t\to -1} t+1 = 0$

f $\lim_{x\to 0} \frac{(x+2)^2-4}{x} = \frac{x^2+4x}{x}$
$= \lim_{x\to 0} x+4 = 4$

g $\lim_{t\to 1} \frac{t^2-1}{t-1} = \frac{(t-1)(t+1)}{t-1}$
$= \lim_{t\to 1} t+1 = 2$

h $\lim_{x\to 9} \sqrt{x+3} = \sqrt{12} = 2\sqrt{3}$

i $\lim_{x\to 0} \frac{x^2-2x}{x} = x-2 = -2$

j $\lim_{x\to 2} \frac{x^3-8}{x-2} = \frac{(x-2)(x^2+2x+4)}{x-2}$
$= \lim_{x\to 2} x^2+2x+4 = 12$

k $\lim_{x\to 2} \frac{3x^2-x-10}{x^2+5x-14} = \frac{(x-2)(3x+5)}{(x-2)(x+7)}$
$= \lim_{x\to 2} \frac{3x+5}{x+7} = \frac{11}{9}$

l $\lim_{x\to 1} \frac{x^2-3x+2}{x^2-6x+5} = \frac{(x-1)(x-2)}{(x-1)(x-5)}$
$= \lim_{x\to 1} \frac{x-2}{x-5} = \frac{1}{4}$

2 a Discontinuities at $x = 3$ and 4, because at $x = 3$, $f(x)$ is not defined, and the right limit of $f(x)$ at $x = 4$ is not equal to $f(4)$. ($x = 1$ is not a discontinuity, although the function is not differentiable there.)

b There is a discontinuity at $x = 7$, because the right limit of $f(x)$ at $x = 7$ is not equal to $f(7)$.

3 a $f(x) = 3x$ if $x \ge 0$, $-2x+2$ if $x < 0$
Discontinuity at $x = 0$: $f(0) = 0$, but
$\lim_{x\to 0^+} f(x) = 0$, $\lim_{x\to 0^-} f(x) = 2$

b $f(x) = x^2+2$ if $x \ge 1$, $-2x+1$ if $x < 1$
Discontinuity at $x = 1$: $f(1) = 3$, but
$\lim_{x\to 1^+} f(x) = 3$, $\lim_{x\to 1^-} f(x) = -1$

c $f(x) = -x$ if $x \le -1$

$f(x) = x^2$ if $-l < x < 0$

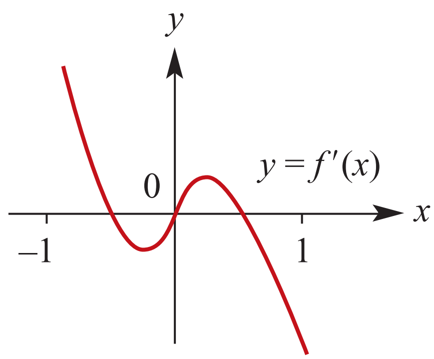
$f(x) = -3x+1$ if $x \ge 0$
Discontinuity at $x = 0$: $f(0) = 1$, but
$\lim_{x\to 0^+} f(x) = 1$, $\lim_{x\to 0^-} f(x) = 0$
$x = -1$ is not a discontinuity, since
$f(-1) = 1$
$\lim_{x\to (-1)^+} f(x) = 1$, $\lim_{x\to (-1)^-} f(x) = 1$

4 $y = \begin{cases} 2; & x < 1 \\ (x-4)^2 - 9; & 1 \le x < 7 \\ x-7; & x \ge 7 \end{cases}$
Discontinuity at $x = 1$: $y(1) = 0$, but
$\lim_{x\to 1^+} y(x) = 0$, $\lim_{x\to 1^+} y(x) = 2$
$x = 7$ is not a discontinuity, since
$y(7) = 0$
$\lim_{x\to 7^+} y(x) = 0$, $\lim_{x\to 7^+} y(x) = 0$

Solutions to Exercise 17I

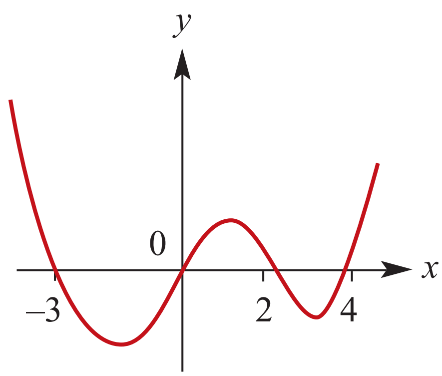
1 a

x	-1	-0.5	0.2	0	0.2	0.5	1
$f'(x)$	$+$	0	$-$	0	$+$	0	$-$

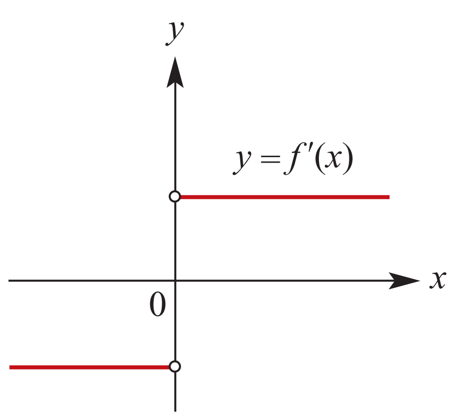


b

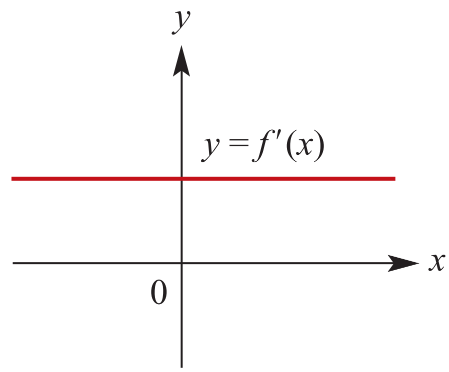
x	-4	-3	-2	0	1	2	3	4	5
$f'(x)$	$+$	0	$-$	0	$+$	0	$-$	0	$+$



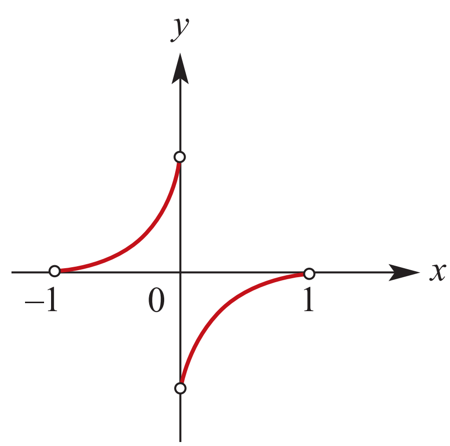
c For $x < 0$, $f'(x) = -k$, $k > 0$
For $x > 0$, $f'(x) = k$, $k > 0$
At $x = 0$, $f'(x)$ is undefined since $f(x)$ is not differentiable at that point.



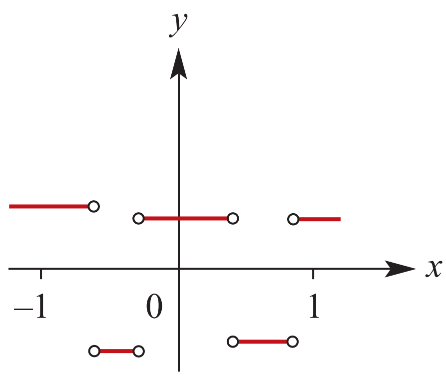
d For all x, $f'(x) = k$, $k > 0$



e $f'(x)$ only exists for
$\{x: -1 < x < 1\}/\{0\}$
For $-1 < x < 0$, $f'(x) < 0$
For $0 < x < 1$, $f'(x) > 0$
At $x = 0$, $f'(x)$ is undefined since $f(x)$ is not differentiable at that point.



f $f'(x)$ is undefined at four points over $[-1, 1]$ and is positive at both ends, alternating $+$ to $-$ between the undefined points:



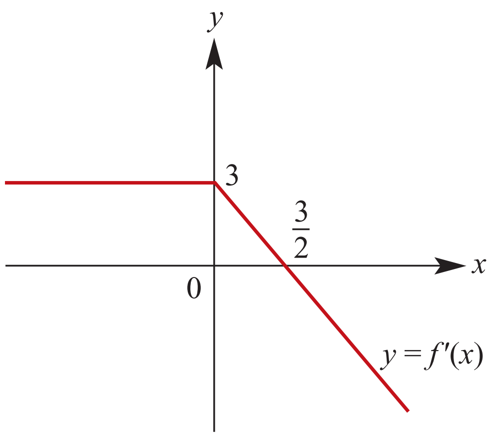
2 $f(x) = -x^2 + 3x + 1$ if $x \geq 0$

$f(x) = 3x + 1$ if $x < 0$

$\therefore\ f'(x) = -2x + 3$ if $x \geq 0$

$f'(x) = 3$ if $x < 0$

$f(x)$ is differentiable at $x = 0$ because both $f(x)$ and $f'(x)$ are continuous at that point.



3 $f(x) = x^2 + 2x + 1$ if $x \geq 1$

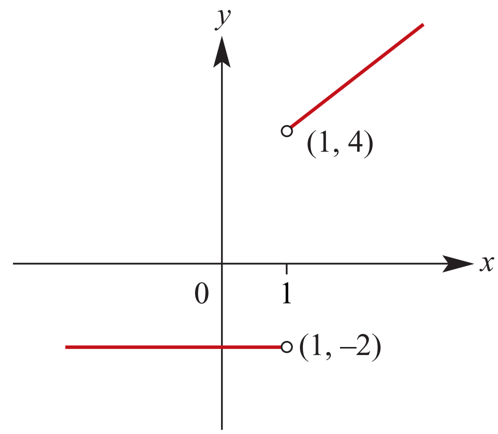
$f(x) = -2x + 3$ if $x < 1$

$\therefore f'(x) = 2x + 2$ if $x > 1$

$f'(x) = -2$ if $x < 1$

$f(x)$ is not differentiable at $x = 0$ because both $f(x)$ and $f'(x)$ are discontinuous at that point.

$\therefore\ f'(x)$ is defined over $R/\{1\}$



4 $f(x) = -x^2 - 3x + 1$ if $x \geq -1$

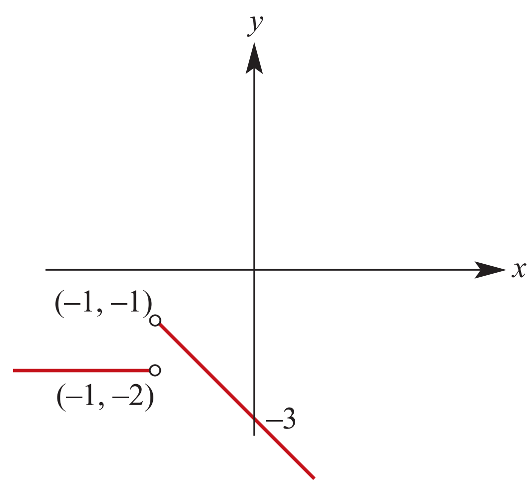
$f(x) = -2x + 3$ if $x < -1$

$\therefore\ f'(x) = -2x - 3$ if $x > -1$

$f(x) = -2$ if $x < -1$

$f(x)$ is not differentiable at $x = -1$ because both $f(x)$ and $f'(x)$ are discontinuous at that point.

$\therefore\ f'(x)$ is defined over $R/\{-1\}$



Solutions to Review: Short-answer questions

1 a $\dfrac{1^3 - 0^3}{1 - 0} = 1$

b $\dfrac{3^3 - 1^3}{3 - 1} = \dfrac{26}{2} = 13$

2 a
$$\frac{f(x+h) - f(x)}{x+h-x} = \frac{3(x+h) + 1 - (3x+1)}{x+h-x} = \frac{3h}{h} = 3$$

$\lim_{h \to 0} 3 = 3$

b
$$\frac{f(x+h) - f(x)}{x+h-x} = \frac{4 - (x+h)^2 - (4 - x^2}{x+h-x} = \frac{4 - x^2 - 2xh - h^2 - 4 + x^2}{h} = \frac{-2xh - h^2}{h} = -2x - h$$

$\lim_{h \to 0} -2x - h = -2x$

c
$$\frac{f(x+h) - f(x)}{x+h-x} = \frac{(x+h)^2 + 5(x+h) - (x^2 + 5x)}{x+h-x} = \frac{x^2 + 2xh + h^2 + 5x + 5h - x^2 - 5x}{h} = \frac{2xh + h^2 + 5h}{h} = 2x + h + 5$$

$\lim_{h \to 0} 2x + h + 5 = 2x + 5$

d
$$\frac{f(x+h) - f(x)}{x+h-x} = \frac{(x+h)^3 + (x+h) - (x^3 + x)}{x+h-x} = \frac{x^3 + 3x^2h + 3xh^2 + x + h - x^3 - x}{h} = \frac{3x^2h + 3xh^2 + h}{h} = 3x^2 + 3xh + 1$$

$\lim_{h \to 0} 3x^2 + 3xh + 1 = 3x^2 + 1$

e
$$\frac{f(x+h) - f(x)}{x+h-x} = \frac{(x+h)^2 + 2(x+h) + 1 - (x^2 + 2x + 1)}{x+h-x} = \frac{x^2 + 2xh + h^2 + 2x + 2h + 1 - x^2 - 2x - 1}{h} = \frac{2xh + h^2 + 2h}{h} = 2x + h + 2$$

$\lim_{h \to 0} 2x + h + 2 = 2x + 2$

f
$$\frac{f(x+h) - f(x)}{x+h-x} = \frac{3(x+h)^2 - (x+h) - (3x^2 - x)}{x+h-x} = \frac{3x^2 + 6xh + 3h^2 - x - h - 3x^2 + x}{h} = \frac{6xh + 3h^2 - h}{h} = 6x + 3h - 1$$

$\lim_{h \to 0} 6x + 3h - 1 = 6x - 1$

3 a $y = 3x^2 - 2x + 6$

$\therefore \dfrac{dy}{dx} = 6x - 2$

b $y = 5, \therefore \dfrac{dy}{dx} = 0$

c $y = 2x(2 - x) = 4x - 2x^2$

$\therefore \dfrac{dy}{dx} = 4 - 4x$

d $y = 4(2x - 1)(5x + 2)$

$= 40x^2 - 4x - 8$

$\therefore \dfrac{dy}{dx} = 80x - 4 = 4(20x - 1)$

e $y = (x + 1)(3x - 2)$

$= 3x^2 + x - 2$

$\therefore \dfrac{dy}{dx} = 6x + 1$

f $y = (x+1)(2-3x)$

$= -3x^2 - x + 2$

$\therefore \dfrac{dy}{dx} = -6x - 1$

4 a $y = -x, \ \therefore \dfrac{dy}{dx} = -1$

b $y = 10, \ \therefore \dfrac{dy}{dx} = 0$

c $y = \dfrac{(x+3)(2x+1)}{4}$

$= \dfrac{1}{2}x^2 + \dfrac{7}{4}x + \dfrac{3}{4}$

$\therefore \dfrac{dy}{dx} = x + \dfrac{7}{4}$

d $y = \dfrac{2x^3 - x^2}{31} = \dfrac{2}{3}x^2 - \dfrac{1}{3}x, \ x \neq 0$

$\therefore \dfrac{dy}{dx} = \dfrac{4}{3}x - \dfrac{1}{3} = \dfrac{1}{3}(4x-1), \ x \neq 0$

e $y = \dfrac{x^4 + 3x^2}{2x^2} = \dfrac{1}{2}x^2 + 3, \quad x \neq 0$

$\therefore \dfrac{dy}{dx} = x, \ x \neq 0$

5 a $y = x^2 - 2x + 1, \ \therefore \dfrac{dy}{dx} = 2x - 2$

At $x = 2, \ y = 1$ and gradient $= 2$

b $y = x^2 - 2x, \ \therefore \dfrac{dy}{dx} = 2x - 2$

At $x = -1, \ y = 3$ and gradient $= -4$

c $y = (x+2)(x-4) = x^2 - 2x - 8$

$\therefore \dfrac{dy}{dx} = 2x - 2$

At $x = 3, \ y = -5$ and gradient $= 4$

d $y = 3x^2 - 2x^3, \ \therefore \dfrac{dy}{dx} = 6x - 6x^2$

At $x = -2, \ y = 28$ and gradient $= -36$

6 a $y = x^2 - 3x + 1, \therefore \dfrac{dy}{dx} = 2x - 3$

$\dfrac{dy}{dx} = 0, \ \therefore 2x - 3 = 0$

$x = \dfrac{3}{2}$

$y\left(\dfrac{3}{2}\right) = \dfrac{9}{4} - \dfrac{9}{2} + 1 = -\dfrac{5}{4}$

Coordinates are $\left(\dfrac{3}{2}, \ -\dfrac{5}{4}\right)$

b $y = x^3 - 6x^2 + 4, \therefore \ \dfrac{dy}{dx} = 3x^2 - 12x$

$\dfrac{dy}{dx} = -12, \therefore 3x^2 - 12x = -12$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0, \ \therefore x = 2$

$y(2) = 8 - 24 + 4 = -12$

Coordinates are $(2, -12)$

c $y = x^2 - x^3, \ \therefore \dfrac{dy}{dx} = 2x - 3x^2$

$\dfrac{dy}{dx} = -1, \ \therefore -3x^2 + 2x + 1 = 0$

$3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$\therefore x = -\dfrac{1}{3}, 1$

$y\left(-\dfrac{1}{3}\right) = \dfrac{4}{27}, \ y(1) = 0$

Coordinates are $\left(-\dfrac{1}{3}, \ \dfrac{4}{27}\right)$ and $(1, 0)$

d $y = x^3 - 2x + 7, \therefore \ \dfrac{dy}{dx} = 3x^2 - 2$

$\dfrac{dy}{dx} = 1, \therefore 3x^2 - 2 = 1$

$3x^2 = 3, \ \therefore x = \pm 1$

$y(-1) = 8; y(1) = 6$

Coordinates are $(-1, 8)$ and $(1, 6)$

e $y = x^4 - 2x^3 + 1, \ \therefore \dfrac{dy}{dx} = 4x^3 - 6x$

$\frac{dy}{dx} = 0, \therefore 4x^3 - 6x^2 = 0$

$2x^2(2x - 3) = 0, \therefore x = 0, \frac{3}{2}$

$y(0) = 1; y\left(\frac{3}{2}\right) = \frac{81}{16} - \frac{27}{4} + 1 = -\frac{11}{16}$

Coordinates are (0, 1) and $\left(\frac{3}{2}, -\frac{11}{16}\right)$

f

$y = x(x - 3)^2 = x^3 - 6x^2 + 9x$

$\therefore \frac{dy}{dx} = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3)$

$\frac{dy}{dx} = 0, \therefore x^2 - 4x + 3 = 0$

$(x - 1)(x - 3) = 0 \therefore x = 1, 3$

$y(1) = 4; y(3) = 0$

Coordinates are (1, 4) and (3, 0)

7 $f(x) = 3(2x - 1)^2 = 12x^2 - 12x + 3$

$\therefore f'(x) = 24x - 12 = 12(2x - 1)$

a $f(x) = 0, \therefore 2x - 1 = 0$

$x = \frac{1}{2}$

b $f'(x) = 0, \therefore 2x - 1 = 0$

$x = \frac{1}{2}$

c $f'(x) > 0, \therefore 2x - 1 > 0$

$x > \frac{1}{2}$

d $f'(x) < 0, \therefore 2x - 1 < 0$

$x < \frac{1}{2}$

e $f'(x) > 0, \therefore 3(2x - 1)^2 > 0$

$\{x : x \in R \backslash \{\frac{1}{2}\}\}$

f $f'(x) = 3, \therefore 24x - 12 = 3$

$24x = 15$

$x = \frac{5}{8}$

8 a $\frac{d}{dx} x^{-4} = -4x^{-5}$

b $\frac{d}{dx} 2x^{-3} = -6x^{-4}$

c $\frac{d}{dx} - \frac{1}{3x^2} = -\frac{1}{3}\frac{d}{dx}x^{-2} = \frac{2}{3x^3}$

d $\frac{d}{dx} - \frac{1}{x^4} = -(-4)x^{-5} = \frac{4}{x^5}$

e $\frac{d}{dx}\frac{3}{x^5} = -15x^{-6} = -\frac{15}{x^6}$

f $\frac{d}{dx}\frac{x^2 + x^3}{x^4} = \frac{d}{dx}x^{-2} + x^{-1} = -\frac{2}{x^3} - \frac{1}{x^2}$

g $\frac{d}{dx}\frac{3x^2 + 2x}{x^2} = \frac{d}{dx}\left(3 + \frac{2}{x}\right) = -\frac{2}{x^2}$

h $\frac{d}{dx}\left(5x^2 - \frac{2}{x}\right) = 10x + \frac{2}{x^2}$

9 $y = ax^2 + bx$

$\therefore \frac{dy}{dx} = 2ax + b$

a Using (1, 1): $a + b = 1$

Gradient = 3: $2a + b = 3$

$\therefore a = 2, b = -1$

b $\frac{dy}{dx} = 0, \therefore 2ax + b = 0$

$\therefore 4x - 1 = 0$

$x = \frac{1}{4}$

$y = 2x^2 - x$

$\therefore y\left(\frac{1}{4}\right) = \frac{1}{8} - \frac{1}{4} = -\frac{1}{8}$

Coordinates are $\left(\frac{1}{4}, -\frac{1}{8}\right)$

10 **a** $\int \frac{1}{2} dx = \frac{x}{2} + c$

b $\int \frac{x^2}{2} dx = \frac{x^3}{6} + c$

c $\int x^2 + 3x\, dx = \frac{x^3}{3} + \frac{3x^2}{2} + c$

d $\int (2x+3)^2 dx = \int 4x^2 + 12x + 9\, dx$

$= \frac{4x^3}{3} + 6x^2 + 9x + c$

e $\int atdt = \frac{1}{2}at^2 + c$

f $\int \frac{1}{3}t^3 dt = \frac{1}{12}t^4 + c$

g

$\int (t+1)(t-2)dt = \int t^2 - t - 2\, dt$

$= \frac{1}{3}t^3 - \frac{1}{2}t^2 - 2t + c$

h

$\int (2-t)(t+1)dt = \int -t^2 - t_2 + 2dt$

$= -\frac{1}{3}t^3 - \frac{1}{2}t^2 + 2t + c$

11 $f'(x) = 2x + 5$

$\therefore\ f(x) = x^2 + 5x + c$

$f(3) = 9 + 15 + c = -1$

$\therefore\quad c = -25$

$f(x) = x^2 + 5x - 25$

12 $f'(x) = 3x^2 - 8x + 3$

$\therefore\ f(x) = x^3 - 4x^2 + 3x + c$

a $f(0) = 0,\ \therefore c = 0$

$\therefore f(x) = x^3 - 4x^2 + 3x$

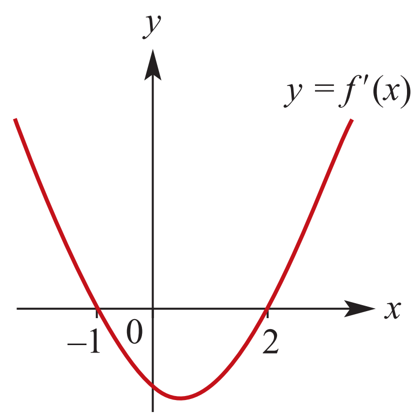
b

$f(x) = 0,\ \therefore x(x-1)(x-3) = x = 0$

$x = 0, 1, 3$

13

x	-2	-1	0	2	3
$f'(x)$	$+$	0	$-$	0	$+$



14 **a** $\{x: h'(x) > 0\} = \{x:\ -1 < x < 4\}$

b $\{x: h'(x) < 0\}$

$= \{x: x < -1\} \cup \{x: x > 4\}$

c $\{x: h'(x) = 0\} = \{-1, 4\}$

Solutions to Review: Multiple-choice questions

1 E $\dfrac{f(2)-f(-2)}{2-(-2)}$

$= \dfrac{2(2)^3+3(2)-2(-2)^3-3(-2)}{2-(-2)}$

$= \dfrac{16+6+16+6}{4}$

$= 11$

2 D $y = x^3 + 4x, \therefore \dfrac{dy}{dx} = 3x^2 + 4$

$\therefore y'(2) = 12 + 4 = 16$

3 B $y = 2x^2$

$\therefore$ chord gradient $= \dfrac{2(1+h)^2 - 2(1)^2}{h}$

$= \dfrac{4h + 2h^2}{h}$

$= 4 + 2h$

4 E $y = 2x^4 - 5x^3 + 2$

$\therefore \dfrac{dy}{dx} = 8x^3 - 15x^2$

5 B $f(x) = x^2(x+1) = x^3 + x^2$

$\therefore f'(x) = 3x^2 + 2x$

$\therefore f'(-1) = 3 - 2 = 1$

6 C $f(x) = (x-3)^2 = x^2 - 6x + 9$

$\therefore f'(x) = 2x - 6$

7 C $y = \dfrac{2x^4 + 9x^2}{3x}$

$= \dfrac{2}{3}x^3 + 3x; x \neq 0$

$\therefore \dfrac{dy}{dx} = 2x^2 + 3; x \neq 0$

8 A $y = x^2 - 6x + 9$

$\therefore \dfrac{dy}{dx} = 2x - 6 \geq 0$ if $x \geq 3$

9 E $y = 2x^4 - 36x^2$

$\therefore \dfrac{dy}{dx} = 8x^3 - 72x = 8x(x^2 - 9)$

Tangent to curve parallel to x-axis where

$8x(x^2 - 9) = 0$

$\therefore x = 0, \pm 3$

10 A $y = x^2 + 6x - 5, \therefore \dfrac{dy}{dx} = 2x + 6$

Tangent to curve parallel to $y = 4x$ where

$\dfrac{dy}{dx} = 2x + 6 = 4$

$\therefore 2x = -2 < \therefore x = -1$

$y(-1) = (-1)^2 + 6(-1) - 5 = -10$

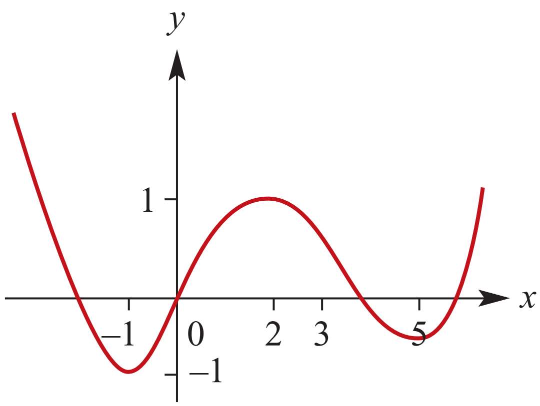
Coordinates are $(-1, -10)$

11 D $y = -2x^3 + 3x^2 - x + 1$

$\therefore \dfrac{dy}{dx} = -6x^2 + 6x - 1$

Solutions to Review: Extended-response questions

1 For $x < -1$, the gradient is negative, becoming less steep as x approaches -1.
For $x = -1$, the gradient is zero.
For $-1 < x < 2$, the gradient is positive, getting steeper as x approaches 0.5 (approximately) then becoming less steep as x approaches 2.
For $x = 2$, the gradient is zero.
For $2 < x < 5$, the gradient is negative, getting steeper as x approaches 4 (approximately) then becoming less steep as x approaches 5.
For $x = 5$, the gradient is zero.
For $x > 5$, the gradient is positive and becoming steeper.



2 $P(x) = ax^3 + bx^2 + cx + d$

At $(0, 0)$, $\quad 0 = 0 + 0 + 0 + d$

$\therefore \quad d = 0$

At $(-2, 3)$, $\quad 3 = a(-2)^3 + b(-2)^2 + c(-2)$

$\therefore \quad 3 = -8a + 4b - 2c \quad (1)$

At $(1, -2)$, $\quad -2 = a(1)^3 + b(1)^2 + c(1)$

$\therefore \quad -2 = a + b + c \quad (2)$

$P'(x) = 3ax^2 + 2bx + c$

At $x = -2$, $P'(x) = 0$, $\quad \therefore 0 = 3a(-2)^2 + 2b(-2) + c$

$\therefore \quad 0 = 12a - 4b + c \quad (3)$

(3) − (2)

$$\begin{aligned} 12a - 4b + c &= 0 \\ -a + b + c &= -2 \end{aligned}$$

(1) + 2 × (2)

$$11a - 5b = 2 \quad (4)$$

$$\begin{aligned} -8a + 4b - 2c &= 3 \\ +2a + 2b + 2c &= -4 \end{aligned}$$

6 × (4) + 5 × (5)

$$-6a + 6b = -1 \quad (5)$$

$$\begin{aligned} 66a - 30b &= 12 \\ + \quad -30a + 30b &= -5 \end{aligned}$$

$$36a = 7$$

∴ $$a = \frac{7}{36} \quad (6)$$

Substitute (6) into (5) $$-6\left(\frac{7}{36}\right) + 6b = -1$$

∴ $$6b = -1 + \frac{7}{6}$$

∴ $$b = \frac{1}{36} \quad (7)$$

Substitute (6) and (7) into (2) $$-2 = \frac{7}{36} + \frac{1}{36} + c$$

∴ $$\begin{aligned} c &= \frac{-72 - 7 - 1}{36} \\ &= \frac{-80}{36} \\ &= \frac{-20}{9} \end{aligned}$$

Hence $$a = \frac{7}{36}, b = \frac{1}{36}, \ c = \frac{-20}{9}, \ d = 0$$

so $$P(x) = \frac{7}{36}x^3 + \frac{1}{36}x^2 - \frac{20}{9}x$$

3 a

$$y = \frac{1}{5}x^5 + \frac{1}{2}x^4$$

$$\frac{dy}{dx} = x^4 + 2x^3$$

i When $x = 1$, $\frac{dy}{dx} = 1^4 + 2(1)^3$

$= 1 + 2$

$= 3$

$\therefore$ $\tan\theta = 3$ where θ is the angle required

$\therefore$ $\theta \approx 71.57°$

ii When $x = 3$, $\frac{dy}{dx} = 3^4 + 2(3)^3$

$= 81 + 54$

$= 135$

$\therefore$ $\tan\theta = 135$

$\therefore$ $\theta \approx 89.58°$

b Consider $\frac{dy}{dx} = 32$

which implies $x^4 + 2x^3 = 32$

i.e. $x^4 + 2x^3 - 32 = 0$

The factor theorem gives that $x - 2$ is a factor

$\therefore$ $(x - 2)(x^3 + 4x^2 + 8x + 16) = 0$

i.e. $\frac{dy}{dx} = 32$ when $x = 2$.

So, gradient path is 32 when $x = 2$ km.

4 a $y = 2 + 0.12x - 0.01x^3$

$$\frac{dy}{dx} = 0.12 - 0.03x^2$$

At the beginning of the trail, $x = 0$

$$\therefore \frac{dy}{dx} = 0.12 - 0.03(0)^2 = 0.12$$

Hence, the gradient at the beginning of the trail is 0.12.

At the end of the trail, $x = 3$

$\therefore$ $\frac{dy}{dx} = 0.12 - 0.03(3)^2$

$= 0.12 - 0.27$

$= -0.15$

Hence, the gradient at the end of the trail is -0.15.

b The trail climbs at the beginning and goes downwards at the end, suggesting a peak in between (i.e. for $0 < x < 3$) where the gradient will be zero.

Gradient is zero where $\frac{dy}{dx} = 0$

$\therefore \quad 0.03x^2 = 0.12$

$\therefore \quad x^2 = 4$

$\therefore \quad x = \pm 2$

$\therefore \quad x = 2$ as $0 < x < 3$

At $x = 2$,

$$\begin{aligned} y &= 2 + 0.12(2) - 0.01(2)^3 \\ &= 2 + 0.24 - 0.08 \\ &= 2.16 \end{aligned}$$

From the above, $\frac{dy}{dx} > 0$ for $x < 2$

and $\frac{dy}{dx} < 0$ for $x > 2$

Hence the gradient is zero when $x = 2$, i.e. 2 km from the beginning of the trail, and the height of the pass is 2.16 km.

5 a

$$\begin{aligned} y &= x(x - 2) \\ &= x^2 - 2x \end{aligned}$$

$$\frac{dy}{dx} = 2x - 2$$

At (0, 0)

$$\begin{aligned} \frac{dy}{dx} &= 2(0) - 2 \\ &= -2 \end{aligned}$$

At (2, 0)

$$\begin{aligned} \frac{dy}{dx} &= 2(2) - 2 \\ &= 2 \end{aligned}$$

Geometrically, the angles of inclination between the positive direction of the x-axis and the tangents to the curve at (0, 0) and (2, 0) are supplementary (i.e. add to 180°).

b

$$\begin{aligned} y &= x(x-2)(x-5) \\ &= x(x^2 - 5x - 2x + 10) \\ &= x(x^2 - 7x + 10) \\ &= x^3 - 7x^2 + 10x \\ \frac{dy}{dx} &= 3x^2 - 14x + 10 \end{aligned}$$

At (0, 0) $\frac{dy}{dx} = l$

$$\therefore \quad l = 3(0)^2 - 14(0) + 10 = 10$$

At (2, 0) $\frac{dy}{dx} = m$

$$\begin{aligned} \therefore \quad m &= 3(2)^2 - 14(2) + 10 \\ &= 12 - 28 + 10 \\ &= -6 \end{aligned}$$

At (5, 0) $\frac{dy}{dx} = n$

$$\begin{aligned} \therefore \quad n &= 3(5)^2 - 14(5) + 10 \\ &= 75 - 70 + 10 \\ &= 15 \end{aligned}$$

$$\begin{aligned} \frac{1}{l} + \frac{1}{m} + \frac{1}{n} &= \frac{1}{10} + \frac{1}{-6} + \frac{1}{15} \\ &= \frac{3 - 5 + 2}{30} \\ &= 0 \text{ as required.} \end{aligned}$$

6 a

$$\frac{f(b) + c - (f(a) + c)}{b - a} = \frac{f(b) - f(a)}{b - a} = m$$

b

$$\frac{cf(b) - cf(a)}{b - a} = c\frac{f(b) - f(a)}{b - a} = cm$$

c

$$\frac{(-f(b)) - (-f(a))}{b - a} = -\frac{f(b) - f(a)}{b - a} = -m$$